



**GCE**

**Mathematics B (MEI)**

**H640/01: Pure Mathematics and Mechanics**

Advanced GCE

**Mark Scheme for Autumn 2021**

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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## 1. Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction <b>In this question you must show detailed reasoning</b> appears in the question.

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**2. Subject-specific Marking Instructions for AS/A Level Further Mathematics B (MEI)**

- a a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.

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c The following types of marks are available.

**M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

**A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**

Mark for a correct result or statement independent of Method marks.

**E**

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep\*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
- Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

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f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is **not given** in the paper accept any answer that agrees with the correct value to **2 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.

NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads “3 s.f”

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

Candidates using a value of 9.80, 9.81 or 10 for  $g$  should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold “In this question you must show detailed reasoning”, or the command words “Show” and “Determine. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

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Question		Answer	Marks	AOs	Guidance
1		For example $(-3)^2 = 9 > 2^2 = 4$ and $(-3) < 2$ So Beth is not correct	<b>M1</b>  <b>E1</b> <b>[2]</b>	<b>2.1</b>  <b>2.2a</b>	Stating any pair of numbers where $p^2 > q^2$ and $p < q$ Fully convincing argument – do not allow for only disproving the converse Also accept general statement about a negative number [for $p$ ]
2		Mass of craft is $\frac{5200}{9.8} = [530.612\dots]$ kg Weight on Mars is $3.7 \times \text{mass} = 3.7 \times 530.6122$ $= 1963 \text{ N (1960 to 3s.f.)}$	<b>M1</b>  <b>M1</b> <b>A1</b> <b>[3]</b>	<b>3.1b</b>  <b>1.1b</b> <b>1.1b</b>	Attempt to find the mass Use of 'mg' with Mars gravity Cao
3	(a)	$y > x + 5$  $y \leq 8 - 2x - x^2$	<b>B1</b>  <b>B1</b> <b>[2]</b>	<b>2.5</b>  <b>2.5</b>	Allow interchange of $>$ and $\geq$ or $<$ and $\leq$ for one inequality as long as the direction is correct Both inequalities fully correct. Allow $x + 5 < y \leq 8 - 2x - x^2$ oe
3	(b)	<b>DR</b> Boundary values when $8 - 2x - x^2 = x + 5$ $x^2 + 3x - 3 = 0$  giving $x = \frac{-3 \pm \sqrt{21}}{2}$  From the graph, the line lies below the curve for $\frac{-3 - \sqrt{21}}{2} < x < \frac{-3 + \sqrt{21}}{2}$  or $\left\{ x : x > \frac{-3 - \sqrt{21}}{2} \right\} \cap \left\{ x : x < \frac{-3 + \sqrt{21}}{2} \right\}$	<b>M1</b>  <b>A1</b>  <b>B1</b>  <b>[3]</b>	<b>2.1</b>  <b>2.1</b>  <b>2.1</b>	A correct three term quadratic equation (or inequality) must be seen  Correct roots of the equation soi Must be exact  Shows correct inequality FT their roots  Any method including BC is acceptable for solving the quadratic equation clearly seen in the form $ax^2 + bx + c = 0$ .  Allow B1 for $-3.79 < x < 0.79$ www

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Question			Answer	Marks	AOs	Guidance	
4	(a)	(i)	Sequence is periodic [with period 4]	<b>B1</b> <b>[1]</b>	<b>1.2</b>	Do not allow repeating, recurring etc.	The sequence can also be described as oscillating
4	(a)	(ii)	Total of 200 terms is $50 \times (2 + 3 + 0 + 3) = 400$	<b>B1</b> <b>[1]</b>	<b>1.1b</b>	cao	
4	(b)	(i)	Sequence divergent for Either $b > 1$  or $b \leq -1$	<b>B1</b>  <b>B1</b> <b>[2]</b>	<b>1.1b</b>  <b>1.1b</b>	Allow for one correct inequality  Must have “or” or the union of sets Condone $b < -1$ or $ b  > 1$	Note for $b = 1$ , the <b>sequence</b> is convergent, but the corresponding <b>series</b> is divergent
4	(b)	(ii)	Infinite sum of geometric series with $a = \frac{1}{3}, r = \frac{1}{3}$  $S = \frac{a}{1-r} = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{2}$	<b>M1</b>  <b>A1</b> <b>[2]</b>	<b>1.1b</b>  <b>1.1b</b>	Using the sum of geometric series with $r = \frac{1}{3}$ www	
5	(a)		$F = 50$ Take moments about A (or D) $Fx = 20 \times 30 + 20 \times 10$  OR Take moments about B (or C) $F(30 - x) + 20 \times 10 = 30 \times 30$ $x = \frac{800}{50} = 16$	<b>B1</b>  <b>M1</b>  <b>A1</b> <b>[3]</b>	<b>1.1b</b>  <b>1.1a</b>  <b>1.1b</b>	cao  Forming an equation for moments about any point –allow their value of $F$ used. Allow one incorrect distance or a missing term  Allow 0.16 m if unit given	Do not allow for moment = force
5	(b)		Forces have a resultant moment  which have a clockwise turning effect on the lamina	<b>E1</b>  <b>E1</b> <b>[2]</b>	<b>2.4</b>  <b>2.4</b>	Allow ‘resultant moment’ or ‘not in equilibrium’ soi  Clockwise must be indicated	Do not allow for ‘a resultant force’ on its own



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Question		Answer	Marks	AOs	Guidance
6	(a)	Asymptotes $x = 0$ (or y-axis) $x = \pi$ and $x = 2\pi$	<b>B1</b> <b>[1]</b>	<b>1.2</b>	Must have all three
6	(b)	When $x = \frac{\pi}{3}$ , $y = \frac{2\sqrt{3}}{3}$ When $x = \frac{\pi}{3}$ , $\frac{dy}{dx} = -\operatorname{cosec} \frac{\pi}{3} \cot \frac{\pi}{3} = -\frac{2}{3}$  Equation of the tangent is $y - \frac{2\sqrt{3}}{3} = -\frac{2}{3} \left( x - \frac{\pi}{3} \right)$ When $y = 0$ , $-\frac{2\sqrt{3}}{3} = -\frac{2}{3} \left( x - \frac{\pi}{3} \right)$ giving $x = \frac{\pi}{3} + \sqrt{3}$ <b>(AG)</b>	<b>B1</b> <b>M1</b> <b>A1</b> <b>M1</b> <b>M1</b> <b>A1</b> <b>[6]</b>	<b>1.1b</b> <b>3.1a</b> <b>1.1b</b> <b>2.1</b> <b>2.1</b> <b>2.1</b>	soi; any exact form eg $\frac{2}{\sqrt{3}}$ Uses the derivative when $x = \frac{\pi}{3}$ May be embedded in the tangent equation Uses both their coordinates and their gradient to find the equation of the tangent Substituting $y = 0$ into their tangent Working must be correct and exact throughout  If $y = -\frac{2}{3}x + c$ used, there must be an attempt to find $c$ using both their coordinates and their gradient

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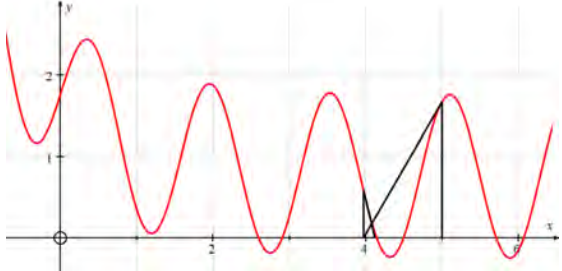
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Question		Answer	Marks	AOs	Guidance
7	(a)	<p><b>DR</b>  Midpoint of AB is (3, 1)  Centre C of the circle is (3, 1)  and radius <math>\sqrt{(7-3)^2 + (-2-1)^2} = 5</math>  So circle is <math>(x-3)^2 + (y-1)^2 = 25</math></p>	<p><b>B1</b>   <b>M1</b>   <b>M1</b>   <b>A1</b>  <b>[4]</b></p>	<p><b>3.1a</b>   <b>3.1a</b>   <b>1.1b</b>   <b>1.1b</b></p>	<p>soi   Attempt to find length of AB, AC or BC  Uses their midpoint and radius (do not allow for diameter used)  Need not be simplified</p>
7	(b)	<p><b>DR</b>  Crosses <math>y = 2x + 5</math> where  <math>(x-3)^2 + (2x+5-1)^2 = 25</math>  <math>5x^2 + 10x = 0</math> giving <math>x = -2, 0</math>  So points are <math>(-2, 1)</math> and <math>(0, 5)</math></p>	<p><b>M1</b>   <b>A1</b>  <b>A1</b>  <b>[3]</b></p>	<p><b>1.1b</b>   <b>1.1b</b>  <b>1.1b</b></p>	<p>Substituting <math>y = 2x + 5</math> and attempting to collect terms oe   Both values correct  Correct y coordinates FT their x-coordinates   Allow for a quadratic solved BC providing it is seen in form  <math>ax^2 + bx = 0</math> or  <math>ay^2 + by + c = 0</math></p>
7	(c)	<p><b>DR</b>  <math>AQ = \sqrt{2}</math> and <math>BQ = \sqrt{7^2 + 7^2} = 7\sqrt{2}</math>  Triangle ABQ has a right angle at Q (angle in a semicircle)  So area of triangle is <math>\frac{1}{2} \times AQ \times BQ</math>   Area = 7</p>	<p><b>M1</b>   <b>M1</b>   <b>A1</b>  <b>[3]</b></p>	<p><b>3.1a</b>   <b>2.1</b>   <b>1.1b</b></p>	<p>Attempt to find two lengths to be used in their area calculation (excluding AB)  Correct method for finding the area   FT their Q   Note <math>\angle QAB = 81.9^\circ</math> and <math>\angle QBA = 8.1^\circ</math></p>

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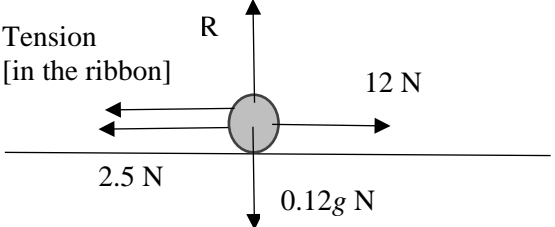
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Question		Answer	Marks	AOs	Guidance								
8	(a)	Karim has a valid argument that there is a root between 5 and 6 because there is a change of sign on his table	<b>E1</b> [1]	<b>2.3</b>	Argues from change of sign that this argument is valid Allow argument is not valid as he does not state that the function is continuous								
8	(b)	There are two roots between 2 and 3 (and/or between 4 and 5) so there is no change of sign in the table	<b>E1</b> [1]	<b>2.3</b>	Allow for a comment that implies changes of sign are missed								
8	(c)	(i) $f'(x) = 4 \cos 4x - e^{-x}$ So N-R formula is $x_{n+1} = x_n - \frac{\sin 4x_n + e^{-x_n} + 0.75}{4 \cos 4x_n - e^{-x_n}}$	<b>M1</b> <b>A1</b> [2]	<b>1.1b</b> <b>1.1b</b>	Attempt to differentiate cao, but condone missing subscripts in the fraction								
8	(c)	(ii) <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td><math>x_0</math></td><td>3</td></tr> <tr><td><math>x_1</math></td><td>2.920853...</td></tr> <tr><td><math>x_2</math></td><td>2.908274...</td></tr> <tr><td><math>x_3</math></td><td>2.907846...</td></tr> </table> <p>The root is 2.908 to 4 s.f.</p>	$x_0$	3	$x_1$	2.920853...	$x_2$	2.908274...	$x_3$	2.907846...	<b>M1</b> <b>A1</b> <b>A1</b> [3]	<b>1.1b</b> <b>1.1b</b> <b>2.2a</b>	Produces at least two iterations Three iterations with correct values either rounded or truncated to at least 3 decimal places Correct to at least 3 s.f. FT their values if their sequence seems to converge (root is 2.907845109 to 10 sf)
$x_0$	3												
$x_1$	2.920853...												
$x_2$	2.908274...												
$x_3$	2.907846...												
8	(d)	(i) 	<b>B1</b> <b>B1</b> [2]	<b>1.1b</b> <b>1.1b</b>	Attempt to draw a tangent at $x = 5$ as far as the $x$ -axis Drawing the second tangent approximately at the point where $x = 3.97$ as far as the $x$ -axis.								
8	(d)	(ii) The start value is close to a stationary point, [so the gradient is very small] and the tangent meets the $x$ -axis far away from the required root The sequence converges to a root, but not the required root	<b>B1</b> <b>B1</b> [2]	<b>2.4</b> <b>2.4</b>	Conveys the idea that the stationary point or the value of the gradient causes the problem Conveys the idea that the wrong root is found								
8	(d)	(iii) Use $x_0$ any value [between 5.28 and 5.85] which is nearer to the required root.	<b>E1</b> [1]	<b>2.4</b>	Allow for a 'starting value between 5 and 6' oe								

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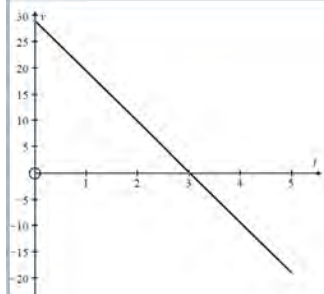
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Question			Answer	Marks	AOs	Guidance	
9	(a)	(i)	Total mass 390 g (0.390 kg) $12 - 4 \times 2.5 = 0.390a$	<b>M1</b> <b>A1</b> [2]	<b>3.3</b> <b>3.3</b>	Newton's second law; condone missing or incorrect resistance or incorrect total mass Fully correct equation need not be simplified	Allow for $T - R = ma$ if the correct substitution of values seen in part 9aii
9	(a)	(ii)	$5.13 \text{ m s}^{-2}$	<b>B1</b> [1]	<b>1.1b</b>	cao	
9	(b)			<b>B1</b> <b>B1</b> <b>B1</b> [3]	<b>1.1b</b> <b>1.1b</b> <b>1.1b</b>	Correct vertical forces 12 N and 2.5 N correctly drawn and labelled with no extra forces Tension in the ribbon correctly drawn and labelled	Allow 'weight' or 120g N oe for 0.12g N Allow 'resistance' for 2.5 N and 'tension' for 12 N providing it is clear that the tension in the string is distinct from the tension in the ribbon
9	(c)		N2L for the head only $12 - 2.5 - T = 0.12a$ OR N2L for the three body sections together $T - 3 \times 2.5 = 0.270a$  $T = 8.88 \text{ N}$	<b>M1</b> <b>A1</b> [2]	<b>1.1b</b> <b>1.1b</b>	N2L with correct mass and their acceleration. Allow missing or incorrect resistance  cao	Allow for mass in grams if same error seen in part (a)  Allow A1 for correct answer from consistent use of grams

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Question		Answer	Marks	AOs	Guidance
10	(a)	Time when $v = 0$ given by $0 = 29.4 - 9.8t$ , so $t = 3$ s	<b>E1</b> <b>[1]</b>	<b>2.1</b>	Using <i>suvat</i> equation(s) leading to correct value for $t$ with $v = 0$  Allow for verifying that $t = 3$ gives $v = 0$ if identified as the maximum point oe
10	(b)		<b>B1</b>  <b>B1</b> <b>[2]</b>	<b>1.1b</b>  <b>1.1b</b>	straight line with negative gradient through either (3, 0) or (0, 29.4)  Both (3, 0) and (0, 29.4) clearly seen Must include negative values of $v$ for $t > 3$
10	(c)	When $t = 5$ , $v = 29.4 - 9.8 \times 5$  $v = -19.6$  Speed is $19.6 \text{ m s}^{-1}$	<b>M1</b>  <b>A1</b>  <b>A1</b> <b>[3]</b>	<b>1.1a</b>  <b>1.1b</b>  <b>1.1b</b>	Using <i>suvat</i> equation(s) leading to a value for $v$ with $t = 5$ . Allow sign errors May be implied by 19.6 seen  FT their negative velocity  If motion from the highest point considered $u = 0$ , $t = 2$ , $g = +9.8$ then $v = 19.6$ is fully correct. Allow M1A1A0 if $29.4 - 9.8 \times 5 = 19.6$ seen
10	(d)	Max height unchanged so $u_y = 29.4 \text{ m s}^{-1}$ Time to max height unchanged, so 3 s	<b>B1</b>  <b>B1</b> <b>[2]</b>	<b>3.1b</b>  <b>3.3</b>	Allow if calculated from $y = 44.1$ m
10	(e)	$u_x \times 3 = 48$ $u = \sqrt{u_x^2 + u_y^2} = \sqrt{16^2 + 29.4^2} = 33.5$ $\tan \alpha = \frac{u_y}{u_x} = \frac{29.4}{16}$ giving $\alpha = 61.4^\circ$	<b>M1</b>  <b>M1</b>  <b>A1</b> <b>[3]</b>	<b>1.1a</b>  <b>1.1b</b>  <b>1.1b</b>	Using (their) $t = 3$ to find $u_x$  Combining their components to find either one of $u$ and $\alpha$  Both values correct

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Question		Answer	Marks	AOs	Guidance
11	(a)	$\frac{dV}{dt} = \frac{k}{x}$ <p>When <math>t = 0</math>, <math>x = 5</math> and <math>\frac{dV}{dt} = 21</math>, so <math>\frac{dV}{dt} = \frac{105}{x}</math></p> $\frac{dV}{dx} = 4\pi x^2,$ <p>so the chain rule gives</p> $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 4\pi x^2 \frac{dx}{dt}$ <p>Hence <math>\frac{dx}{dt} = \frac{1}{4\pi x^2} \times \frac{105}{x} = \frac{105}{4\pi x^3}</math> <b>AG</b></p>	<b>M1</b> <b>A1</b> <b>B1</b> <b>M1</b> <b>A1</b> <b>[5]</b>	<b>3.3</b> <b>3.3</b> <b>2.1</b> <b>2.1</b> <b>3.3</b>	Expresses inverse proportionality with a constant Evaluating $k$ , oe (may be done later) may be embedded in chain rule Use of the chain rule Convincing argument
11	(b)	$\int 4\pi x^3 dx = \int 105 dt$ $\pi x^4 = 105t + c$ <p>When <math>t = 0</math>, <math>x = 5</math> so <math>c = 625\pi</math></p> $\text{When } t = 120 \quad x = \sqrt[4]{\frac{105}{\pi} \times 120 + 625} = 8.25 \text{ cm}$	<b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b> <b>A1</b> <b>[5]</b>	<b>3.1a</b> <b>1.1b</b> <b>3.3</b> <b>3.3</b> <b>3.4</b>	Separating the variables Condone missing $+c$ here Using initial conditions Correct value for $c$ cao
11	(c)	As $t$ gets very large, the volume gets very large so the balloon will get beyond the maximum it can be without bursting and so burst.	<b>E1</b>  <b>[1]</b>	<b>3.5b</b>	Conveys the idea that $t \rightarrow \infty \Rightarrow V \rightarrow \infty$ or $x \rightarrow \infty$ Indicates a practical problem with very large volume

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12		Normal reaction $mg \cos 15^\circ$	<b>B1</b>	<b>3.1b</b>	Correct normal reaction	Only allow $0.4mg$ if it is clear that $mg$ is their normal reaction and not just weight
		Max friction $\mu N = 0.4mg \cos 15^\circ$	<b>M1</b>	<b>1.1b</b>	Attempt to evaluate friction FT their normal reaction.	
		Resolve down the slope	<b>B1</b>	<b>1.1b</b>	Correct component of weight seen ( $2.536m$ )	
		$mg \sin 15^\circ - F = ma$ $mg \sin 15^\circ - 0.4mg \cos 15^\circ = ma$ giving $a = -1.25 \text{ m s}^{-2}$	<b>M1</b>	<b>3.1b</b>	All terms present; allow sign errors, sin/cos interchange for weight and their $F$	
			<b>A1</b>	<b>1.1b</b>	Correct equation ( $a$ need not be explicitly evaluated here)	
		Using $v^2 = u^2 + 2as$ $0^2 = 1.2^2 + 2 \times (-1.25)s$ giving $s = 0.576 \text{ m}$	<b>M1</b>	<b>3.1b</b>	Use of <i>suvat</i> equation(s) leading to a value for $s$ using $v = 0$	
			<b>A1</b>	<b>1.1b</b>	FT their negative $a$	
			<b>[7]</b>			

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Question		Answer	Marks	AOs	Guidance	
13	(a)	$\mathbf{r} = \int (\mathbf{v}) dt = \int (3t^2\mathbf{i} + 7\mathbf{j}) dt$ $= t^3\mathbf{i} + 7t\mathbf{j} + \mathbf{c}$ When $t = 0, \mathbf{r} = -\mathbf{i} + 2\mathbf{j} = \mathbf{c}$ So $\mathbf{r} = (t^3 - 1)\mathbf{i} + (7t + 2)\mathbf{j}$ When $t = 2,$ $\mathbf{r} = (2^3 - 1)\mathbf{i} + (7 \times 2 + 2)\mathbf{j} = 7\mathbf{i} + 16\mathbf{j}$  $\text{distance} = \sqrt{7^2 + 16^2} = \sqrt{305}$ $\text{distance} = 17.5 \text{ m}$	<b>M1*</b>  <b>M1 dep*</b> <b>M1 dep*</b> <b>A1</b>  <b>M1</b> <b>A1</b> <b>[6]</b>	<b>3.4</b>  <b>1.1b</b>  <b>3.1b</b>  <b>1.1b</b>  <b>3.1b</b> <b>3.2a</b>	Attempt to integrate both components; condone missing +c  Using initial conditions  Using $t = 2$ to find position vector or values for $x$ and $y$ Accept vector form or two clear components.  Using Pythagoras FT their components	
13	(b)	using $x = t^3 - 1, y = 7t + 2$  Substitute $t = \frac{y-2}{7}$ into equation for $x$  $x = \left(\frac{y-2}{7}\right)^3 - 1 \quad \mathbf{AG}$	<b>M1</b>  <b>M1 dep</b>  <b>A1</b> <b>[3]</b>	<b>3.1a</b>  <b>1.1b</b>  <b>1.1b</b>	Extracting equations for $x$ and $y$ from their displacement vector Attempt to eliminate $t$  cao	Equivalent form $y = 7(x+1)^{\frac{1}{3}} + 2$ for M1M1A0
13	(c)	$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 6t\mathbf{i}$ When $t = 2, \mathbf{a} = 12\mathbf{i}$  The force must be in that direction, so $\mathbf{F} = 48\mathbf{i} = m\mathbf{a}$  So $m = 4 \text{ kg}$	<b>M1*</b>  <b>M1*</b>  <b>M1*</b> <b>A1 dep*</b> <b>[4]</b>	<b>3.1b</b>  <b>3.4</b>  <b>3.1b</b>  <b>1.1b</b>	Must be vector acceleration  Evaluating when $t = 2$  Newton's second law in vector form, or in $x$ -direction only cao	$a = 12$ is sufficient here  If their $\mathbf{a}$ has two non-zero components, allow for dividing 48 by the magnitude of their $\mathbf{a}$



**OCR (Oxford Cambridge and RSA Examinations)**  
**The Triangle Building**  
**Shaftesbury Road**  
**Cambridge**  
**CB2 8EA**

**OCR Customer Contact Centre**

**Education and Learning**

Telephone: 01223 553998

Facsimile: 01223 552627

Email: [general.qualifications@ocr.org.uk](mailto:general.qualifications@ocr.org.uk)

[www.ocr.org.uk](http://www.ocr.org.uk)

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